CSE 125
Discrete Mathematics

Nazia Sultana Chowdhury
nazia.nishat1971@gmail.com

The Growth of Functions

- Determining how fast an algorithm can solve a problem as the size of the input grows.
- Comparing the efficiency of two different algorithms for solving the same problem.

Describing Growth of Functions

- Big-O Notation
- Big-Omega
- Big-Theta Notation

Big-O Notation

Let f and g be functions from the set of integers or the set of real numbers to the set of real numbers.

- $f(x)$ is $O(g(x))$ if there are constants C and k such that

  $|f(x)| \leq C|g(x)|$ whenever $x > k$.

Big-O Notation

- Describes the long-term growth rate of functions.
- Doesn't care about constants.
- Gives an upper bound.

$f(n)$, in terms of $O(g(n))$?

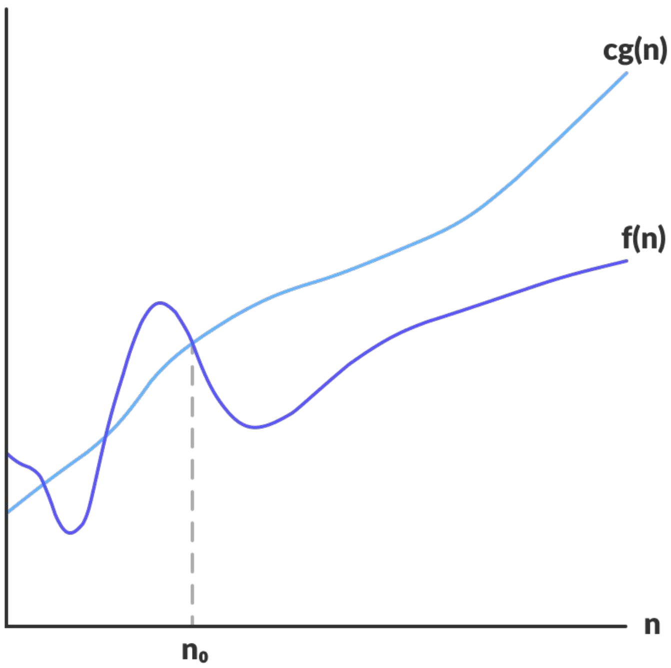
Here, $f(n) = n^2 + 2n$

$$f(n) \leq c\, g(n)$$

$\Rightarrow n^2 + 2n \leq$ ________

$\Rightarrow n^2 + 2n \leq c \cdot n^2$

$\Rightarrow n^2 + 2n \leq 3n^2$, for $c = 3$ and $n \geq 1$



$f(n) = O(g(n))$

Big-Omega

- Let f and g be functions from the set of integers or the set of real numbers to the set of real numbers.
- $f(x)$ is $\Omega(g(x))$ if there are positive constants C and k such that $|f(x)| \geq C|g(x)|$ whenever $x > k$.
- Lower Bound.

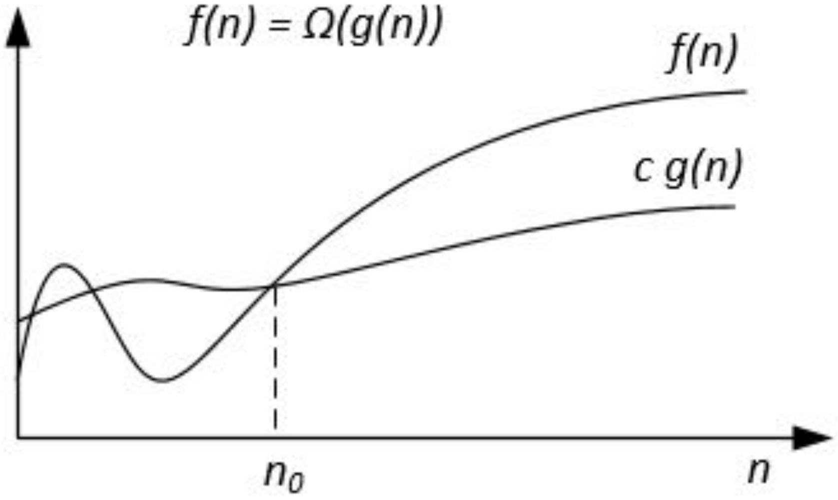
$f(n)$ in terms of $\Omega(g(n))$?

$$f(n) \geq c\, g(n)$$

$$\Rightarrow n^2 + 2n \geq \_\_\_\_\_$$

$$\Rightarrow n^2 + 2n \geq c\, n^2$$

$$\Rightarrow n^2 + 2n \geq n^2, \quad \text{for } c = 1 \text{ and } n \geq 1$$


$f(n) = \Omega(g(n))$
$f(n)$
$c\, g(n)$
n_0
n

Big-Theta Notation

Let f and g be functions from the set of integers or the set of real numbers to the set of real numbers.

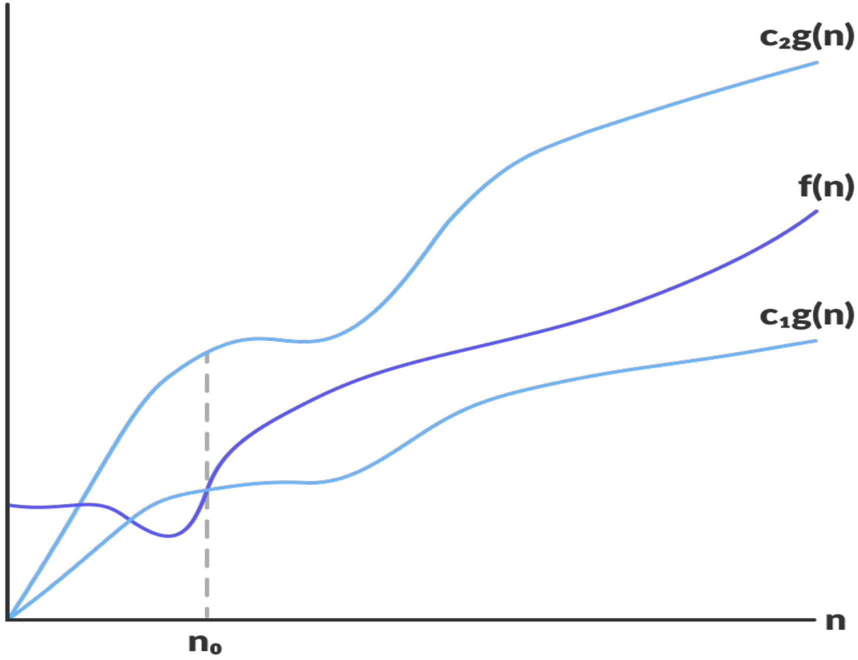
$f(x)$ is $\Theta(g(x))$ if

- $f(x)$ is $O(g(x))$ and
- $f(x)$ is $\Omega(g(x))$.

Big-Theta

$$c_1\, g(n) \le f(n) \le c_2\, g(n)$$

$$\Rightarrow \quad n^2 - 2 \le n^2 + 2n \le 3n^2$$



$f(n) = \Theta(g(n))$